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Technical Notes

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Comparison of Navier–Stokes and Direct Simulation Monte Carlo Predictions with Separation

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I. Introduction

AN IMPORTANT feature of hypersonic flight vehicle flow fields is shock/boundary layer interaction, and in laminar flow this often provokes a sizable region of separation followed by reattachment. Accordingly, CFD tools are under development for predicting the details of such flows, combined with validating experiments and checks against basic theoretical relations [1].

One aspect of these studies seeks to establish the overlap between the direct simulation Monte Carlo (DSMC) codes designed to treat highly rarefied flows and Navier–Stokes codes appropriate to continuum flows. In particular, one may ask how far into the continuum regime can the DSMC approach be extended if it is finely enough gridded to treat the flow within interaction-induced separation zones. The present paper addresses this question by directly comparing a Navier–Stokes analytical theory of the separation and reattachment streamline angles with the DSMC predictions of Moss and Bird [2], while using Bird's code called DS2V [3]. Such comparisons provide a severe test of the code because these angles depend sensitively on the local streamwise pressure and skin friction gradients.

The paper first derives from a local analysis of the Navier–Stokes equations relationships for the separation and reattachment angles in steady laminar flow on cooled axisymmetric bodies including wall slip effects. Detailed comparisons of their predictions against DSMC code results are then presented for typical separation zones found on the hypersonic double-cone (Fig. 1) and hollow cylinder-flare body geometries. The results support the use of a finely gridded DSMC code to resolve the detailed physics of near continuum flows.

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II. Theoretical Background

The key theoretical aspect of the present study is a local analysis of the basic Navier–Stokes equations that yields a relationship governing the separation and reattachment streamlines associated with a separation zone in a laminar flow. The original derivation was carried out for 2-D incompressible flows by Oswatitsch [4]; using a double Taylor's series expansion of the flow properties in x and y about the separation point, he obtained from the exact continuity and momentum equations the following relationship (Fig. 2a):

$$\tan\theta_S = \lim_{x,y\to 0}\left(\frac{v}{u}\right) = -3\left(\frac{\mathrm{d}\tau_w/\mathrm{d}x}{\mathrm{d}p_w/\mathrm{d}x}\right)_s \tag{1}$$

which fully includes local viscous-inviscid interaction effects in the indicated derivatives. It can be readily shown that Eq. (1) also applies to the reattachment angle of an already-separated flow (Fig. 2b). Equation (1), involving as it does the ratio of the local wall shear stress gradient to the corresponding streamwise pressure gradient, can thus provide a highly sensitive check on the detailed accuracy of any purely CFD-type prediction scheme that deals with separated flow including a code based on a DSMC approach.

More recently, Inger [5] showed that Eq. (1) is in fact also applicable to arbitrarily compressible, nonadiabatic flows on axisymmetric as well as 2-D bodies, and so it may be confidently applied to the type of hypersonic double-cone flows that have been the subject of intensive computational and experimental study.

III. Experimental Validation

Before proceeding further, it is appropriate to briefly review the experimental evidence that supports Eq. (1). In the case of a typical high speed laminar flow involving a ramp-provoked separation bubble (Fig. 3), detailed wall pressure, skin friction and streamline pattern measurements were subsequently examined by Inger [6] as a means of checking Eq. (1): Table 1 shows the resulting comparisons of the values of θ_S and θ_R calculated from Eq. (1) using the experimental $\mathrm{d}\tau_w/\mathrm{d}x$ and $\mathrm{d}p_w/\mathrm{d}x$ data with the values measured directly from the observed streamline pattern of Fig. 3c, where it can be seen that they are in good agreement.

Turning to the case of low speed flows, Inger [6] showed that Eq. (1) can be converted into an equivalent expression that permits comparison with the large body of experimental data on separation angles obtained by Dobbinga et al. [7]. Thus, by using the well-known Thwaites boundary layer integral method [8] to evaluate $\mathrm{d}\tau_w/\mathrm{d}x$, Inger obtained the following relationship of θ_S to the momentum thickness Reynolds number at separation:

$$\tan\theta_S = B[(Re_{\theta^*})_S]^{-1} \tag{2}$$

where the nondimensional constant lies in the range $15 \le B \le 20$. A comparison of the prediction of Eq. (2) against the aforementioned data, shown in Fig. 4, exhibits excellent agreement over a very wide range of Reynolds numbers.

The aforementioned experimental validations of Eq. (1) and the fact it is based on the basic Navier–Stokes equations establish confidence in its use as a tool to check the detailed local accuracy of CFD predictions.

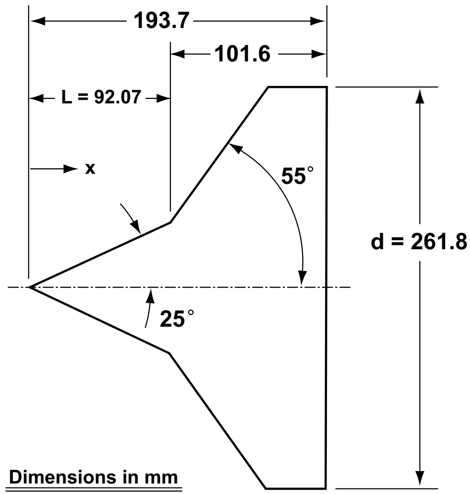

Fig. 1 Double-cone model used for computational [2] and experimental [1] studies of separating hypersonic flows.

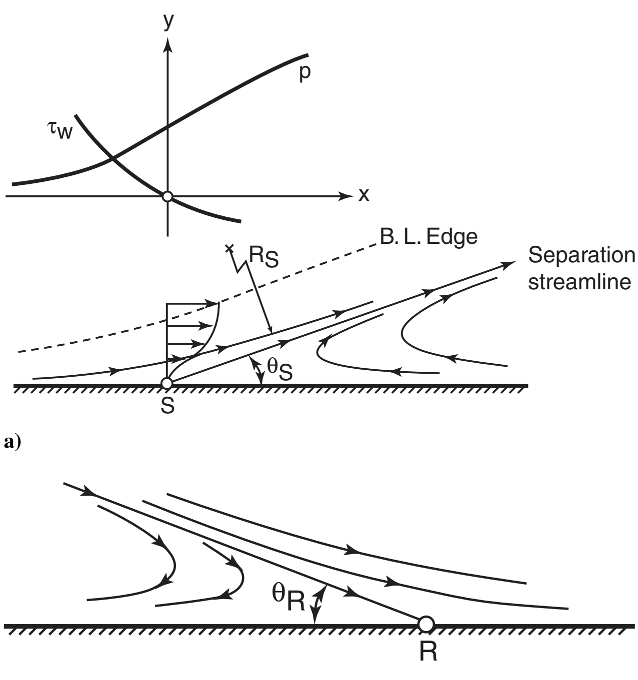

Fig. 2 Schematic of the flow field for a) vicinity of separation and b) near reattachment.

IV. Comparisons of Direct Simulation Monte Carlo Predictions in Hypersonic Separated Flow

The DSMC computational approach has been applied in detail by Moss and Bird [2] to the flow field around the benchmark hypersonic double-cone configuration of Fig. 1. In particular, their predictions for events in the vicinity of corner separation for a typical case are shown in Fig. 5a (pressure and skin friction distributions) and in Fig. 5b (streamlines pattern). Correspondingly, Fig. 6 illustrates the direct comparison of the separation streamline angle predicted by Eq. (1) on the basis of Fig. 5a with the value of θ_S obtained from Fig. 5b: it is seen that the two are in excellent agreement, being within a degree of each other. Other cases, similarly examined, show comparable accuracy. It would thus appear that a properly finely scaled DSMC code can correctly resolve even such detailed local interactive physics.

Regarding the reattachment region pertaining to Fig. 6, a comparable detailed resolution of the DSMC-predicted streamline pattern (Fig. 7) reveals some irregularity not present at separation; this is attributed to the extremely low reversed flow velocities upstream that tend to promote inaccuracy within the DSMC computational cells very close to the surface. Nevertheless, the reattachment property values obtained from Fig. 5 when used in Eq. (1) yield a value $\theta_S = 29$ deg that still agrees reasonably well with the averaged value of 32 deg discerned from Fig. 7.

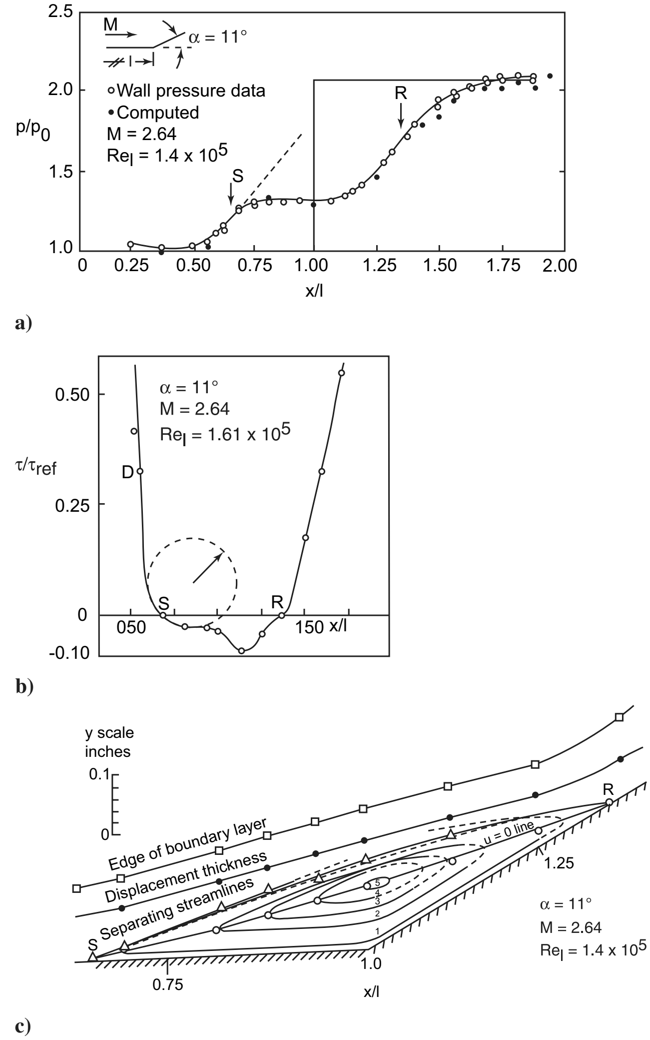

Fig. 3 Ramp-provoked separation in a supersonic laminar flow [5]: a) interactive wall pressure distribution; b) wall shear stress; c) streamline pattern in separation bubble.

V. Wall Slip Effects

Under some lower Reynolds number conditions examined in the aforementioned double-cone studies, the mean free path near the body surface is large enough to yield discernible wall slip effects (Fig. 8). It is therefore of interest to examine the effect of such slip on the nature of this basic separation angle relationship of Eq. (1) and how this in turn may alter the comparisons discussed above.

Accordingly, the derivation of Eq. (1) has been extended to now include the presence of a tangential slip velocity u_w given by the classical kinetic theory/Navier Stokes result

Table 1 Comparison of theoretical vs experimental separation bubble angles (data of Fig. 4)

Parameter	Theory	Measured
$\tan\theta_S$	0.074	0.107
$\tan\theta_R$	0.091	0.112

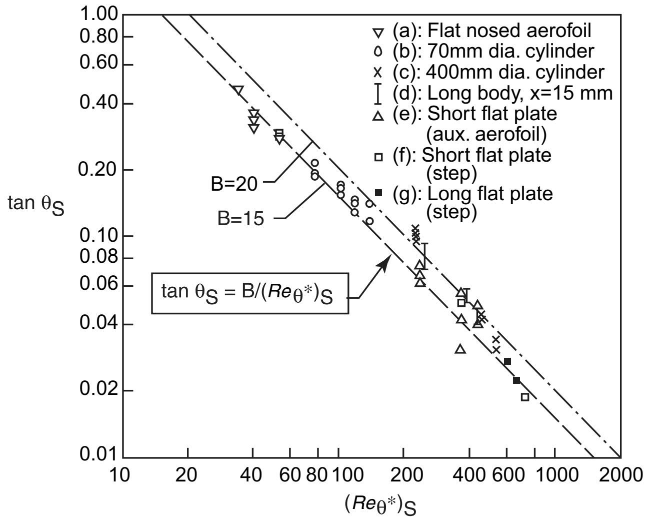

Fig. 4 Separation streamline angles in low speed laminar flows: comparison of theory vs data [7].

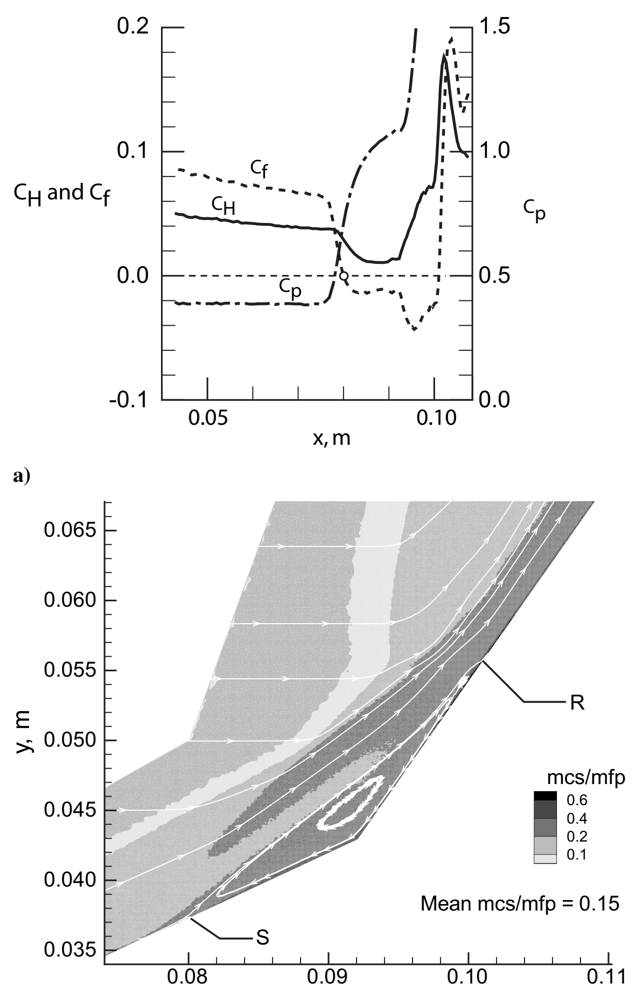

Fig. 5 Results of a detailed DSMC simulation of the separated flow region on a hypersonic double cone [2]: a) pressure, skin friction, and heat transfer distributions; b) streamline pattern along with contours of simulation resolution (ratio of the mean collision separation distance to the local mean free path, mcs/mfp).

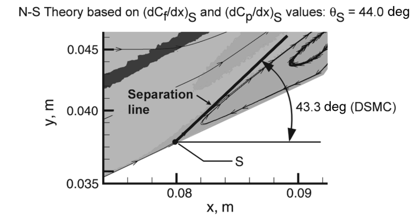

Fig. 6 Qualitative comparison of DSMC and Navier–Stokes predictions for the slip effect on separation streamline shape.

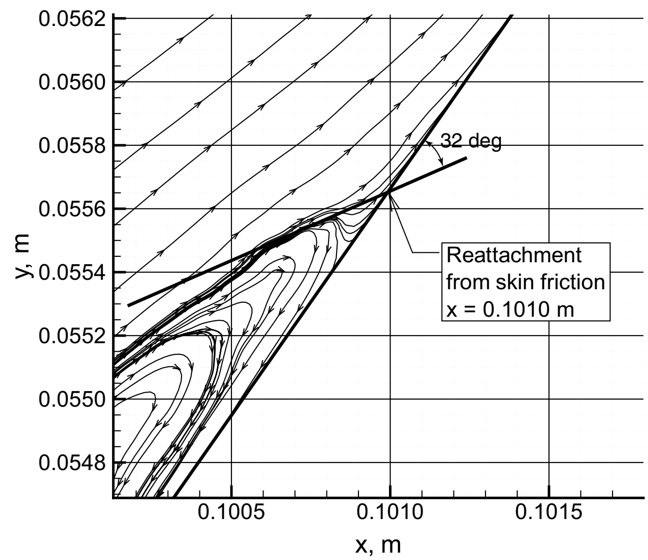

Fig. 7 Computational features (stream lines) of the reattachment region for the case of Fig. 6.

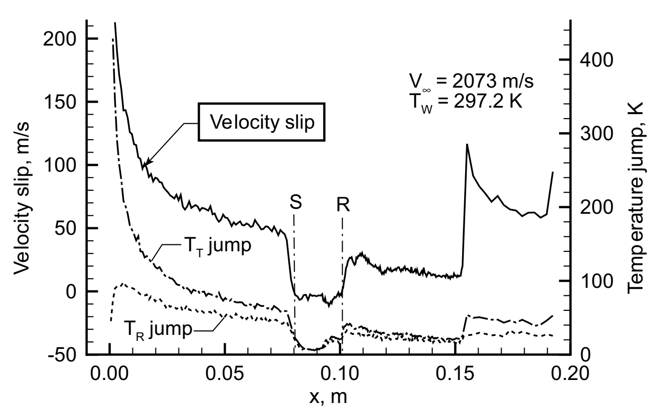

Fig. 8 DSMC-predicted wall slip effects along the hypersonic double-cone separation zone [2] along with the temperature jump values for the translational and rotational temperatures.

$$u_w(x) = \lambda_{wS}\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_w = \frac{\lambda_{wS}}{\mu_{wS}}\tau_w(x) \tag{3}$$

where λ_{wS} is the wall mean free path at separation. We note in connection with Eq. (3) that it predicts zero slip right at separation or reattachment with a negative value in the reversed flow region; see Fig. 8. When expressed in terms of the zero slip value "θ_{S0}" given by Eq. (1), the result of such a generalized local analysis of the compressible Navier–Stokes equations yields the following closed form expression describing the separation streamline shape $y_{Sl}(x)$:

$$x \tan\theta_{S0} = \frac{(1 - C_1\lambda_{wS})y_{Sl}^2}{2\lambda_{wS} + (1 - C_2\lambda_{wS})y_{Sl}} \tag{4a}$$

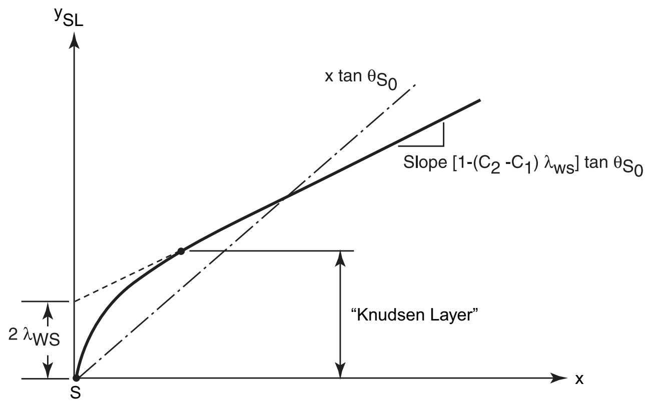

Fig. 9 Schematic of the wall slip effect on separation streamline shape predicted by a local Navier–Stokes theory.

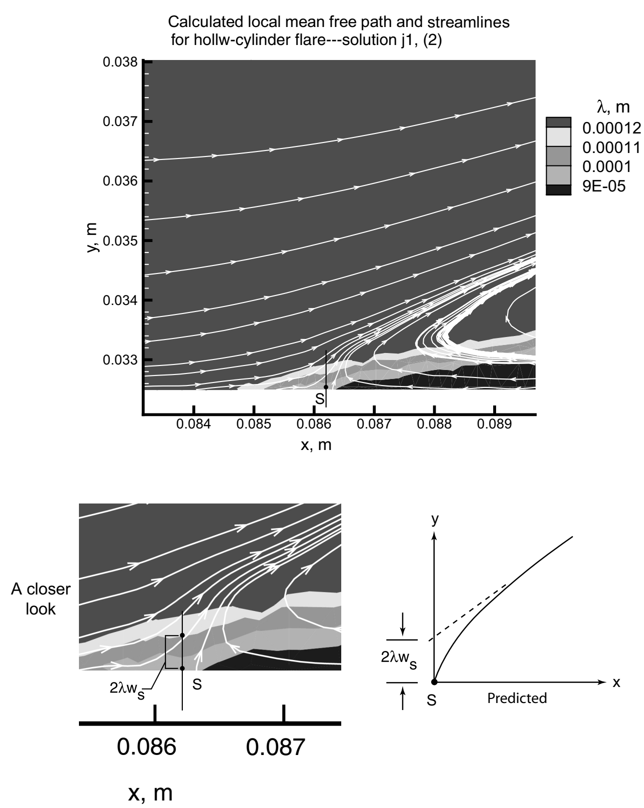

Fig. 10 Qualitative comparison of DSMC simulation and Navier–Stokes theory prediction for separation streamline shape.

where

$$\tan\theta_{S0} = -3\left(\frac{\mathrm{d}\tau_w/\mathrm{d}x}{\mathrm{d}p_w/\mathrm{d}x}\right)_S \tag{4b}$$

and the constants C_1, C_2 contain explicit compressibility and heat transfer effects as follows:

$$C_1 = \left(\frac{\mathrm{d}p_w/\mathrm{d}x}{p_w}\right)_S \tag{4c}$$

$$C_2 = \left(\frac{\mathrm{d}T/\mathrm{d}y}{T}\right)_{wS} \tag{4d}$$

A plot of $y_{Sl}(x)$ from Eq. (4a), illustrated in Fig. 9, shows that slip causes the separation streamlines to leave the wall normal to its surface, thereby introducing a highly curved shape within a "Knudsen zone" of approximately $2\lambda_{wS}$ in size downstream of the separation point. Within this zone, Eq. (4a) yields the very small scale behavior

$$y_{Sl}(x) \approx (\sqrt{2\lambda_{wS} x \tan\theta_{S0}}) \qquad (y \ll y_{wS}) \tag{5}$$

whereas far outside the zone where $y_{Sl} \gg \lambda_{wS}$, Eq. (4b) yields the continuum-type linear behavior

$$y_{Sl}(x) \approx [1 + (C_1 - C_2)\lambda_{wS}] x \tan\theta_{S0} \tag{6}$$

which is seen to involve a very slight slip-induced slope reduction compared with the classical Oswatitsch result of Eq. (1).

For the sake of at least a qualitative comparison, one of the more rarefied, low Reynolds number cases computed in [2] (hollow cylinder-flare model, case j1), wherein the wall mean free path is large enough to be of the same order as the small scale features of the separation zone, has been selected for detailed examination (Fig. 10). It is seen that the slip-induced separation streamline curvature predicted by Eq. (4a) in Fig. 9 agrees with the small scale behavior discerned in the DSMC-predicted flow pattern.

VI. Summary and Conclusions

The paper first derives from a local analysis of the Navier–Stokes equations relationships for the separation and reattachment angles in steady laminar flow on cooled axisymmetric bodies including wall slip effects. Detailed comparisons of their predictions against DSMC code results are then presented for typical separation zones found on the hypersonic double-cone and hollow cylinder-flare body geometries. The results support the use of a finely gridded DSMC code to resolve the detailed physics of near continuum flows.

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